Chapter 9 Review
Properties of Transformations
CHAPTER 9: Properties of Transformations

9.1 Translate Figures and Use Vectors
9.2 Use Properties of Matrices
9.3 Perform Reflections
9.4 Perform Rotations
9.5 Apply Compositions of Transformations
9.6 Identify Symmetry
9.7 Identify and Perform Dilations
Performing Congruence and Similarity Transformation

Translation
Translate a figure right or left, up or down.

Reflection
Reflect a figure in a line.
Performing Congruence and Similarity Transformation

**Rotation**

Rotate a figure about a point.

![Rotation Diagram](image)

**Dilation**

Dilate a figure to change the size but not the shape.

![Dilation Diagram](image)
Making Real-World Connections to Symmetry and Tessellations

Line symmetry

4 lines of symmetry

Rotational symmetry

$90^\circ$ rotational symmetry
Applying Matrices and Vectors in Geometry

- You can use matrices to represent points and polygons in the coordinate plane.

- You can use matrix addition to represent translations, matrix multiplication to represent reflections and rotations, and scalar multiplication to represent dilations.

- You can also use vectors to represent translations.
REVIEW KEY VOCABULARY:

- image
- preimage
- isometry
- vector
  - initial point, terminal point
  - horizontal component
  - vertical component
- component form
- matrix
- element
- dimensions
REVIEW KEY VOCABULARY:

• line of reflection
• center of rotation
• angle of rotation
• glide reflection
• composition of transformations
• line symmetry
• line of symmetry
• rotational symmetry
• center of symmetry
• scalar multiplication
1. Copy and complete: A(n) \underline{isometry} is a transformation that preserves lengths.

2. Draw a figure with exactly one line of symmetry.
3. **WRITING**  Explain how to identify the dimensions of a matrix. Include an example with your explanation.

Count the number of rows, $n$, and columns, $m$. The dimensions are $n \times m$.

\[
\begin{bmatrix}
D & E & F \\
2 & 3 & 5 \\
1 & 6 & 4
\end{bmatrix}
\]

is $2 \times 3$
VOCABULARY EXERCISES

Match the point with the appropriate name on the vector.

4. $T$ B
5. $H$ A

A. Initial point
B. Terminal point
The vector is $\overrightarrow{EF}$. From initial point $E$ to terminal point $F$, you move 4 units right and 1 unit down. So, the component form is $\langle 4, -1 \rangle$. 

\textbf{9.1 TRANSLATE FIGURES AND USE VECTORS}

Name the vector and write its component form:
6. The vertices of ΔABC are A(2, 3), B(1, 0), and C(-2, 4). Graph the image of ΔABC after the translation \((x, y) \rightarrow (x + 3, y - 2)\).

\[
\begin{bmatrix}
2 & 1 & -2 \\
3 & 0 & 4 \\
\end{bmatrix}
+\begin{bmatrix}
3 & 3 & 3 \\
-2 & -2 & -2 \\
\end{bmatrix}
=\begin{bmatrix}
5 & 4 & 1 \\
1 & -2 & 2 \\
\end{bmatrix}
\]
7. The vertices of ΔDEF are D(-6, 7), E(-5, 5), and F(-8, 4). Graph the image of ΔDEF after the translation using the vector \((-1, 6)\).

\[
\begin{bmatrix}
-6 & -5 & -8 \\
7 & 5 & 4 \\
6 & 6 & 6
\end{bmatrix} + \begin{bmatrix}
-1 & -1 & -1 \\
6 & 6 & 6
\end{bmatrix}
= \begin{bmatrix}
-7 & -6 & -9 \\
13 & 11 & 10
\end{bmatrix}
\]
9.2 Use Properties of Matrices

Add \[
\begin{bmatrix}
-9 & 12 \\
5 & -4
\end{bmatrix}
+ \begin{bmatrix}
20 & 18 \\
11 & 25
\end{bmatrix}.
\]

These two matrices have the same dimensions, so you can perform the addition. To add matrices, you add corresponding elements.

\[
\begin{bmatrix}
-9 & 12 \\
5 & -4
\end{bmatrix}
+ \begin{bmatrix}
20 & 18 \\
11 & 25
\end{bmatrix}
= \begin{bmatrix}
-9 + 20 & 12 + 18 \\
5 + 11 & -4 + 25
\end{bmatrix}
= \begin{bmatrix}
11 & 30 \\
16 & 21
\end{bmatrix}
\]
9.2 Use Properties of Matrices

Find the image matrix that represents the translation of the polygon. Then graph the polygon and its image.

\[
\begin{bmatrix}
A & B & C \\
2 & 8 & 1 \\
4 & 3 & 2 \\
\end{bmatrix};
\]

5 units up and 3 units left

\[
\begin{bmatrix}
2 & 8 & 1 \\
4 & 3 & 2 \\
\end{bmatrix} + \begin{bmatrix}
-3 & -3 & -3 \\
5 & 5 & 5 \\
\end{bmatrix} = \begin{bmatrix}
-1 & 5 & -2 \\
9 & 8 & 7 \\
\end{bmatrix}
\]
9.2 Use Properties of Matrices

Find the image matrix that represents the translation of the polygon. Then graph the polygon and its image.

\[
\begin{bmatrix}
D & E & F & G \\
-2 & 3 & 4 & -1 \\
3 & 6 & 4 & -1
\end{bmatrix};
\]

2 units down
## 9.3 Perform Reflections

### Coordinate Rules for Reflections

- If \((a, b)\) is reflected in the \(x\)-axis, its image is the point \((a, -b)\).
- If \((a, b)\) is reflected in the \(y\)-axis, its image is the point \((-a, b)\).
- If \((a, b)\) is reflected in the line \(y = x\), its image is the point \((b, a)\).
- If \((a, b)\) is reflected in the line \(y = -x\), its image is the point \((-b, -a)\).
The vertices of \( \triangle MLN \) are \( M(4, 3), L(6, 3), \) and \( N(5, 1) \). Graph the reflection of \( \triangle MLN \) in the line \( p \) with equation \( x = 2 \).

Point \( M \) is 2 units to the right of \( p \), so its reflection \( M' \) is 2 units to the left of \( p \) at \((0, 3)\). Similarly, \( L' \) is 4 units to the left of \( p \) at \((-2, 3)\) and \( N' \) is 3 units to the left of \( p \) at \((-1, 1)\).
Graph the reflection of the polygon in the given line: $x = 4$
Graph the reflection of the polygon in the given line:

\[ y = 3 \]
9.3 Perform Reflections

Graph the reflection of the polygon in the given line:

\[ y = x \]
9.4 Perform Rotations

**Coordinate Rules for Rotations about the Origin**

When a point \((a, b)\) is rotated counterclockwise about the origin, the following are true:

1. For a rotation of 90°, \((a, b) \to (-b, a)\).
2. For a rotation of 180°, \((a, b) \to (-a, -b)\).
3. For a rotation of 270°, \((a, b) \to (b, -a)\).
9.4 Perform Rotations

Find the image matrix that represents the 90° rotation of ABCD about the origin.

The polygon matrix for \( ABCD \) is \[
\begin{bmatrix}
-2 & 1 & 2 & -3 \\
4 & 4 & 2 & 2
\end{bmatrix}
\].

Multiply by the matrix for a 90° rotation.

\[
\begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
-2 & 1 & 2 & -3 \\
4 & 4 & 2 & 2
\end{bmatrix}
= \begin{bmatrix}
-4 & -4 & -2 & -2 \\
-2 & 1 & 2 & -3
\end{bmatrix}
\]
9.4 Perform Rotations

Find the image matrix that represents the given rotation of the polygon about the origin. Then graph the polygon and its image.

\[
\begin{bmatrix}
3 & 4 & 1 \\
0 & 5 & -2
\end{bmatrix}; 180^\circ
\]

\[
\begin{bmatrix}
-3 & -4 & -1 \\
0 & -5 & 2
\end{bmatrix}
\]
9.4 Perform Rotations

Find the image matrix that represents the given rotation of the polygon about the origin. Then graph the polygon and its image.

\[
\begin{bmatrix}
-1 & 3 & 5 & -2 \\
6 & 5 & 0 & -3
\end{bmatrix}; 270^\circ
\]

\[
\begin{bmatrix}
6 & 5 & 0 & -3 \\
1 & -3 & -5 & 2
\end{bmatrix}
\]
9.5 Apply Compositions of Transformations

The vertices of $\triangle ABC$ are $A (4, -4)$, $B (3, -2)$, and $C (8, -3)$. Graph the image of $\triangle ABC$ after the glide reflection.

Translation: $(x, y) \rightarrow (x, y + 5)$

Reflection: in the $y$-axis

Begin by graphing $\triangle ABC$. Then graph the image $\triangle A'B'C'$ after a translation of 5 units up. Finally, graph the image $\triangle A''B''C''$ after a reflection in the $y$-axis.
Graph the image of $H (-4, 5)$ after the glide reflection.

Translation: $(x, y) \rightarrow (x + 6, y - 2)$

Reflection $x = 3$
9.5 Apply Compositions of Transformations

Graph the image of H (-4, 5) after the glide reflection.

Translation: \((x, y) \rightarrow (x - 4, y - 5)\) \hspace{1cm} \text{Reflection } y = x

\[
\begin{pmatrix}
-4 \\
5
\end{pmatrix} \quad \begin{pmatrix}
-8 \\
0
\end{pmatrix} \quad \begin{pmatrix}
0 \\
-8
\end{pmatrix}
\]
9.6 Identify Symmetry

Determine whether the rhombus has line symmetry and/or rotational symmetry. Identify the number of lines of symmetry and/or the rotations that map the figure onto itself.

The rhombus has two lines of symmetry. It also has rotational symmetry, because a $180^\circ$ rotation maps the rhombus onto itself.
9.6 Identify Symmetry

Determine whether the rhombus has line symmetry and/or rotational symmetry. Identify the number of lines of symmetry and/or the rotations that map the figure onto itself.

Line symmetry, no rotational symmetry; one
9.6 Identify Symmetry

Determine whether the rhombus has line symmetry and/or rotational symmetry. Identify the number of lines of symmetry and/or the rotations that map the figure onto itself.

No line symmetry, rotational symmetry; 180° about the center.
9.6 Identify Symmetry

Determine whether the rhombus has line symmetry and/or rotational symmetry. Identify the number of lines of symmetry and/or the rotations that map the figure onto itself.

19.

line symmetry, rotational symmetry; two, 180° about the center.
9.7 Identify and Perform Dilations

Quadrilateral $ABCD$ has vertices $A(1, 1)$, $B(1, 3)$, $C(3, 2)$, and $D(3, 1)$. Use scalar multiplication to find the image of $ABCD$ with its center at the origin and a scale factor of 2.

To find the image matrix, multiply each element of the polygon matrix by the scale factor.

$$\begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 3 & 2 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 & 6 & 6 \\ 2 & 6 & 4 & 2 \end{bmatrix}$$
9.7 Identify and Perform Dilations

Find the image matrix that represents a dilation of the polygon centered at the origin with the given scale factor. Then graph the polygon and its image.

\[
\begin{bmatrix}
2 & 4 & 8 \\
2 & 4 & 2 \\
\end{bmatrix}; k = \frac{1}{4}
\]

\[
\begin{bmatrix}
\frac{1}{2} & 1 & 2 \\
\frac{1}{2} & 1 & \frac{1}{2} \\
\end{bmatrix}
\]
Identify and Perform Dilations

Find the image matrix that represents a dilation of the polygon centered at the origin with the given scale factor. Then graph the polygon and its image.

\[
\begin{bmatrix}
-1 & 1 & 2 \\
-2 & 3 & 4
\end{bmatrix}; k = 3
\]

\[
\begin{bmatrix}
-3 & 3 & 6 \\
-6 & 9 & 12
\end{bmatrix}
\]
HOMEWORK

Chapter 9 Practice Test
P632: 1-16